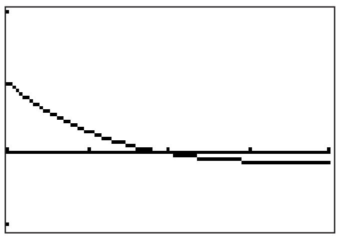
42. Method I: By the Fundamental Theorem of Calculus

$g'(x) = (5 + 4x - x^2)(2^{-x}) = -(x - 5)(x + 1)(2^{-x})$ On the interval (3,5), $g' > 0$ so g is increasing (3,5) on, so I is true. On the interval (5,7), $g' < 0$ so g is decreasing on (5,7), so II is false. Use a graphing calculator to find $g(7) = \int_3^7 (5 + 4t - t^2)(2^{-t})\, dt = 0.562$, so III is false.

Method II: Graph the integrand $(5 + 4x - x^2)(2^{-x})$ in a suitable window such as [3,7] by [−1,2] (See figure). This is the derivative of g and from the graph it is clear that the derivative is positive on (3,5) indicating g is increasing, so I is true.



The derivative is negative on (5,7) indicating that g is decreasing, so II is false.

$g(7)$ is the net area between the axis and the graph. Since there is more area above the axis than below, $g(7) > 0$ and III is false.

The correct choice is (A).

43. Using the disk method, the volume is

$$\pi k^2 \int_0^5 (x-5)^4\, dx = 2500\pi$$

Integrating and solving for k,

$$\pi k^2 \left. \frac{(x-5)^5}{5} \right|_0^5 = 2500\pi$$

$$\pi k^2 (5^4) = 2500\pi$$

$$k^2 = 4 \text{ or } k = 2$$

The correct choice is (E).

44. a is a constant equal to $f(2)$ which is negative, as seen on the graph.

b will correspond to $f'(2) = 0$, since at $x = 2$, f has a relative minimum.

c corresponds to $\frac{f''(2)}{2!}$ which is greater than 0, since $f(x)$ is concave upward at $x = 2$.

Alternate Solution: The Taylor polynomial is a parabola whose vertex is at the point $(2, a)$ which is also the relative minimum point of the function graphed. Therefore $a < 0$, $b = 0$. Since the curve is concave upward at this point, the parabola must open upward, therefore $c > 0$.

The correct choice is (A).

45. Euler's Method, the tangent line approximation, calculates the approximate value of y as

$$y \approx f(x) + f'(3)\,\Delta x = -2 + \left(-\frac{3^2}{-2}(-0.3)\right) = -2 - 1.35 = -3.35$$

The correct choice is (E).