

10. $\int_1^{\infty} x^{-\frac{5}{4}} dx$ is an improper integral.

$$\begin{aligned}\int_1^{\infty} x^{-\frac{5}{4}} dx &= \lim_{t \rightarrow \infty} \int_1^t x^{-\frac{5}{4}} dx = \lim_{t \rightarrow \infty} \left(\frac{x^{-\frac{1}{4}}}{-\frac{1}{4}} \right) \Big|_1^t \\ &= \lim_{t \rightarrow \infty} \left(\frac{t^{-\frac{1}{4}}}{-\frac{1}{4}} - \frac{1^{-\frac{1}{4}}}{-\frac{1}{4}} \right) \\ &= 0 - \frac{1}{-\frac{1}{4}} = 4\end{aligned}$$

The correct choice is (A).

11. Since the function is concave down, the first derivative must always decrease.

In table (A): $\frac{f(5) - f(4)}{5 - 4} = -4$ so by the Mean Value Theorem there must be a value, c_1 , between $x = 4$ and $x = 5$ where $f'(c_1) = -4$. Also $\frac{f(6) - f(5)}{6 - 5} = -4$ so there must be a value c_2 , between $x = 5$ and $x = 6$ where $f'(c_2) = -4$. Thus, the derivative goes from -4 to -3 (given) to -4 . Since the derivative is not decreasing throughout the interval, (A) cannot be the correct table.

Doing a similar analysis on the other tables, the derivatives, in the same order, are

(B) $-2, -3, -4$

(C) $-2, -3, -1$

(D) $-2, -3, -2$

(E) $-5, -3, -1$

This eliminates (C), (D) and (E). The remaining choice (B) is the only one that *could* be the table of values for f .

Although what happens between the values given in the table is unknown, the slopes in (B) clearly indicate a decrease from -2 to -4 and therefore (B) is the best possible choice. In all of the other four choices it is clear that the slopes do not continually decrease.

The correct choice is (B).

12. $y' = \frac{k(k+x) - (kx+8)}{(k+x)^2}$, or

$$y' = \frac{k^2 - 8}{(k+x)^2}$$

Since $y = x + 4$ is the line tangent to y at $x = -2$, its slope is $y' = 1$.

By substituting $x = -2$ and $y'(-2) = 1$,

$$1 = \frac{k^2 - 8}{(k-2)^2} \Rightarrow k^2 - 8 = (k-2)^2 \text{ or } k^2 - 8 = k^2 - 4k + 4, \text{ and } k = 3.$$

The correct choice is (D).

13. Notice that the subintervals are not of equal width. Multiplying the function on the right side of each subinterval times the width of that subinterval gives

$$7(1) + 11(3) + 12(2) + 8(1) = 72$$

The correct choice is (E).

14. The series is a geometric series with a ratio of $\left(-\frac{\pi}{3}\right)$.

Since the absolute value of the ratio is larger than 1, the series will not converge.

The correct choice is (E).

15. By the First Derivative Test, for a function to have a minimum value the derivative must change sign from negative to positive, this is choice (D). Choices (A) and (C) do not have enough information to justify a maximum or minimum. Choice (B) is wrong because for a maximum, the derivative must change from positive to negative. For choice (E) to be a correct justification by the Second Derivative Test, you also need to know that $f'(3) = 0$.

The correct choice is (D).

16. At a point of inflection where the graph changes from concave up to concave down the *second* derivative ($g'' = f'$) changes from positive to negative. This occurs where the *first* derivative ($g' = f$) changes from increasing to decreasing. The value is $x = 5$. Since f has a relative maximum at $x = 5$, f' changes from positive to negative and thus g changes from concave up to concave down.

The correct choice is (D).