

6a. $g(4.5) = 13.5$. This is the area under the graph of f in the first quadrant between $x = 0$ and $x = 4.5$.

$g'(4.5) = f(4.5) = 0$; $g''(4.5) = f'(4.5) = -2$, the slope of the line segment containing $(4.5, 0)$.

$$\begin{aligned} \text{6b. Average value of } f \text{ equals } \frac{1}{5 - (-3)} \int_{-3}^5 f(x) dx &= \frac{1}{8} \left[\int_{-3}^3 f(x) dx + \int_3^5 f(x) dx \right] \\ &= \frac{1}{8} [27 + 2] = \frac{29}{8} \end{aligned}$$

The two integrals are evaluated by finding the signed areas under the graph of f .

6c. There is a point of inflection where f , the derivative of g , has a maximum or minimum. This occurs only at $x = 5$, where f changes from decreasing to increasing, or where f' changes from negative to positive.

6d. The function g increases from $x = -3$ to $x = 4.5$, then decreases until $x = 7$, and then increases again until $x = 9$. There is a maximum at $(4.5, 13.5)$ (from part (a)). There is an endpoint maximum at $x = 9$. Use the signed area to find $g(9)$:

$$\begin{aligned} g(9) &= g(4.5) + \int_{4.5}^9 f(x) dx \\ &= 13.5 - 0.25 \end{aligned}$$

The endpoint maximum occurs at $(9, 13.25)$.