

SAMPLE EXAMINATION III

Answers to Multiple-Choice Questions

1. The area of the region is represented by $\int_1^3 (3x^2 + 2x) dx$.

$$\int_1^3 (3x^2 + 2x) dx = x^3 + x^2 \Big|_1^3 = (27 + 9) - (1 + 1) = 36 - 2 = 34$$

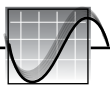
The correct choice is (B).

2. Rewrite the function without the absolute value signs:

$$f(x) = \begin{cases} -x, & x < 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$$

$$\begin{aligned} \text{Then } \int_{-4}^2 f(x) dx &= \int_{-4}^0 (-x) dx + \int_0^2 x dx \\ &= -\frac{x^2}{2} \Big|_{-4}^0 + \frac{x^2}{2} \Big|_0^2 \\ &= (0 - (-8)) + (2 - 0) \\ &= 10 \end{aligned}$$

The correct choice is (D).



3. The units of a definite integral or a Riemann sum are the units of the dependent variable, $C(s)$, multiplied by the units of the independent variable, ds , in this case

$$(\text{gallons/mile}) \times (\text{miles/hour}) = \text{gallons/hour}$$

The correct choice is (B).

4. Given: $\frac{dr}{dt} = 2$

Find: $\frac{dV}{dt}$ when $r = 10$

Since the volume of a sphere is $V = \frac{4}{3}\pi r^3$, differentiate both sides of the equation with respect to t .

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\text{or } \frac{dV}{dt} = 4\pi(10)^2(2) = 800\pi$$

The correct choice is (D).

5. Since the laser beam changes direction at $t = 2$, the total distance traveled by the laser beam is calculated using two integrals. For motion to the left: $\left| \int_1^2 (t^2 - 4) dt \right|$, and for motion to the right:

$$\int_2^3 (t^2 - 4) dt.$$

$$\left| \int_1^2 (t^2 - 4) dt \right| = \left| \left(\frac{t^3}{3} - 4t \right) \Big|_1^2 \right| = \left| \left(\frac{8}{3} - 8 \right) - \left(\frac{1}{3} - 4 \right) \right| = \frac{5}{3}$$

$$\int_2^3 (t^2 - 4) dt = \left(\frac{t^3}{3} - 4t \right) \Big|_2^3 = \left(\frac{27}{3} - 12 \right) - \left(\frac{8}{3} - 8 \right) = \frac{7}{3}$$

Therefore the total distance traveled by the laser beam is 4 feet.

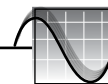
The correct choice is (A).

6. By the Fundamental Theorem of Calculus: $\int_1^4 f'(x) dx = f(4) - f(1) = 2 - (-5) = 7$

The correct choice is (D).

7. The given limit is the derivative of $g(x)$ at $x = 3$. Since $g'(3)$ is negative, the function must be decreasing at $x = 3$.

The correct choice is (A).



8. Let $u = x^2 + 1$

$$du = 2x dx \Rightarrow x dx = \frac{1}{2} du$$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln |u| = \frac{1}{2} \ln (x^2 + 1) \Big|_1^3 = \frac{1}{2} (\ln 10 - \ln 2)$$

Since $\ln 10 - \ln 2 = \ln \left(\frac{10}{2}\right) = \ln 5$, therefore, $\int_1^3 \frac{x}{x^2+1} dx = \frac{1}{2} \ln 5$.

The correct choice is (C).

9. Using the properties of logarithms and the Chain Rule:

$$\begin{aligned} \frac{d}{dx} \ln \left(\frac{1}{x^2-1} \right) &= \frac{d}{dx} (\ln 1 - \ln (x^2 - 1)) \\ &= \frac{d}{dx} (-\ln (x^2 - 1)) \\ &= -\frac{2x}{x^2-1} = \frac{2x}{1-x^2} \end{aligned}$$

The correct choice is (A).

10. Method 1: The average rate of change is given by $\frac{f(b)-f(a)}{b-a} = \frac{e^{(9)}-e^{(-3)}}{3-(-3)} = 0$. The instantaneous rate of change is the derivative $f'(x) = 2xe^{(x^2)}$. Solving $2xe^{(x^2)} = 0$ gives only one value of $x = 0$.

Method 2: The instantaneous rate of change is given by the derivative of f : $f'(x) = 2xe^{(x^2)}$. Since the derivative is always increasing ($f''(x) = (4x^2 + 2)e^{(x^2)} > 0$) it will equal every real number exactly once. Therefore whatever the average value is, it will equal it once.

The correct choice is (B).