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"TYPE" PROBLEMS

The AP Calculus exams contain fresh carefully thought out often cleaver questions. This is especially true for the free-response questions. The topics and style of the questions are similar from year to year. By style I mean whether the stem presents the given information as an equation, a graph or a table of values. Most of the topics can be and have been presented in each of the styles. With that in mind this chapter will discuss the common types of questions on the exams. Some will be discussed based on their style and other by the calculus topic being tested.

Some of the questions test topics that are usually found in one or several contiguous sections of most textbook. These include the area-volume questions, differential equations, and in BC the parametric equation-vector question, and the power series question.

The others tend to draw questions from diverse parts of the book taught at different times of the year, yet based on the same stem. Students need to be aware of this and be ready to shift gears in the middle of the question. They may not be used to this since textbook questions tend to be about the topic in that section and not drawn from previous sections.

Using released exam questions during the year and especially as part of the review at the end of the year will help students draw their knowledge together and do their best on the exam. Another way of developing this skill in students is to make all tests and quizzes cumulative from the beginning of the year. Always include a few from previous units on each test.

Keep in mind that each part of a free-response question can be re-written as a multiple-choice question. You can also use them as shorter open end questions on your tests.

What follows is a discussion of the 10 types of questions.

Type 1: Rate & Accumulation

These questions are often in a "real" context with a lot of words describing a situation in which some things are changing. There are usually two rates given acting in opposite ways. These are also known as "in-out" questions.Students are asked about the change that the rates produce over some time interval either separately or together. The rates may be given in an equation, a graph or a table.

The question almost always appears on the calculator allowed part of the free-response exam and the rates are often fairly complicated functions. If they are on the calculator allowed section, students should store the equations in the equation editor of their calculator and use their calculator to do any equation solving, integration, or differentiation that may be necessary.

Shorter questions on this concept appear in the multiple-choice sections. As always, look over as many questions of this kind from past exams.

What should students be able to do?

- Be ready to read and apply; often these problems contain a lot of writing which needs to be carefully read and interepreted.
- Recognize that the word "rate" means that a derivative is given even though the word "derivative" may not appear.
- Recognize a rate from the units given without the words "rate" or "derivative."
- Find the change in an amount by integrating the rate. The integral of a rate of change gives the amount of change (FTC): $\int_a^b f'(t)\,dt = f(b) - f(a)$.
- Find the final amount by adding the initial amount to the amount found by integrating the rate. The final accumulated amount is $f(t) = f(t_0) + \int_{t_0}^{t} f'(x)dx$, where $t = t_0$ is the initial time, and $f(t_0)$ is the initial amount.
- Understand the question. It is often not necessary to as much computation as it seems at first.
- Use FTC to differentiate a function defined by an integral. Remember, the integrand is the derivative of the integral.
- Explain the meaning of a derivative or its value in terms of the context of the problem.
- Explain the meaning of a definite integral or its value in terms of the context of the problem.
- Justify your answer.

AP Questions

(FR) **2007** AB *2*
(FR) **2009** AB *3*
(FR) **2010** AB *1*
BC *1*
(MC) **2012** AB *81*
BC *81*
(FR) **2013** AB *1*
BC *1*

- Store functions in their calculator recall them to do computations on their calculator.
- If the rates are given in a table, be ready to approximate an integral using a Riemann sum or by trapezoids.
- Do a max/min or increasing/decreasing analysis.

These topics are discussed in chapters 7, 14 and 20 of this book.

Type 2: Particle Moving on a Line

These questions may give the position, the velocity, or the acceleration along with an initial condition. Students may be asked about the motion of the particle: its direction, when it changes direction, when it is farthest left or right, when it turns around, how far it travels its position at a certain time, etc. Speed, the absolute value of velocity, is also a common topic.

The particle may be a "particle," a person, a car, etc. The position, velocity, or acceleration may be given as an equation, a graph or a table. There are a lot of different things students may be asked to find. While particles don't really move in this way, the question is a versatile way to test a variety of calculus concepts.

What should students be able to do?

AP Questions

(FR) **2003** AB *5*
(FR) **2006** AB *4*
(FR) **2008** AB *4*, BC *4*
(FR) **2009** AB *1*, BC *1*
(MC) **2012** AB *16, 28, 79, 83, 89*
BC *2, 89*
(FR) **2013** AB *2*

- Solve an initial value differential equation problems: given the velocity (or acceleration) with initial condition(s) find the position (or velocity).
- Distinguish between position at some time, and the total distance traveled during the time (displacement).
  - The total distance traveled is the definite integral of the speed (absolute value of velocity): $\int_a^b |v(t)|dt$
  - The net distance (displacement) is the definite integral of velocity: $\int_a^b v(t)dt$
  - The final position is the initial position plus the net change in distance from $x = a$ to $x = t$: $s(t) = s(a) + \int_a^t v(x)dx$ Notice that this is an accumulation function equation.
- Find the speed at a particular time.
- Find average speed, velocity, or acceleration. (Average rate of change, average value of a function)
- Determine the speed and whether it is increasing or decreasing.
  - If at some time, the velocity and acceleration have the *same* sign then the speed is increasing.
  - If they have *different* signs the speed is decreasing.
  - If the velocity graph is moving away from (towards) the t-axis the speed is increasing (decreasing).
- Use a difference quotient to approximate derivative from a graph or table. (Show a quotient even if the denominator is 1.)
- Approximate velocity or acceleration from a graph of table.

- Approximate an integral using a Riemann sum or trapezoid sum with values from a table.
- Determine units of measure.
- Interpret meaning of a derivative or a definite integral in context of the problem.
- Justify your answer.

These topics are discussed in chapter 9 with ideas from chapters 11, 14 and 20 of this book.

Type 3: Interpreting Graphs

The long name is "Here's the graph of the derivative, tell me things about the function."

Most often students are given the graph identified as the derivative of a function. There is no equation given and it is not expected that students will write the equation; rather, students are expected to determine important features of the function or its graph directly from the graph of the derivative. They may be asked for the location of extreme values, intervals where the function is increasing or decreasing, concavity, etc. They may be asked for function values at points, found by using the areas of the regions on the derivative's graph.

The graph may be given in context and student will be asked about that context. The graph may be identified as the velocity of a moving object and questions will be asked about the motion and position.

Less often the function's graph may be given and students will be asked about its derivatives.

What should students be able to do?

- Read information about the function from the graph of the derivative. This may be approached using derivative techniques or techniques using the definite integral.
- Find and justify extreme values (1st and 2nd derivative tests, closed interval test, etc.)
- Find where the function is increasing or decreasing.
- Find points of inflection.
- Write an equation of tangent line.
- Evaluate Riemann or Trapezoidal sums from geometry of the graph or from a table
- Evaluate integrals from areas of regions on the graph.
- Understand that the function, $g(x)$, maybe defined by an integral where the given graph is the graph of the integrand, $f(t)$. So students should know that if $g(x) = g(a) + \int_a^x f(t)\,dt$, then $g'(x) = f(x)$ and $g''(x) = f'(x)$
- Justify your answer.

AP Questions

(FR) **2005** AB *5*
(FR) **2010** AB *5*
(FR) **2011** AB *4*, BC *4*
(MC) **2012** AB *15, 17, 76, 80, 85, 87*
BC *15, 18, 76, 78, 80, 88*
(FR) **2012** AB *3*, BC *3*
(FR) **2013** AB *4*, BC *4*